ESTIMATING HOSPITAL MEAN LENGTH OF STAY FROM CENSORED DATA

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1. Introduction

This paper reports on a study of hospital census data undertaken at the University of South Florida Medical Center. It was conducted under the auspices of a joint project between the Departments of Psychiatry and Mathematics, the purpose of which was to bring together statistical and medical professionals, in order to investigate important and timely problems of interest to the latter. The primary area of concern was the evaluation of mental health care delivery. Two Tampa hospitals, St. Joseph's Hospital Mental Health Center and the Veterans Administration Hospital, generously made data available to the project.

The primary data base consists of several complete series dating from the opening of St. Joseph's Mental Health Center in late May 1971 to March 1975. These series include the daily census total (that is, number of beds occupied), number of admissions daily, number of discharges daily, and the length of stay and age of each discharged patient. In addition, the data for the first three series is extended through March 1977, for a total range of some 1,960 days. A time series analysis of this data, which should be read simultaneously with the present paper, can be found elsewhere in this volume (Smith, Welch, and Holland, 1977). We deal here with the analysis of the length of stay data, with the purpose of investigating procedures for estimating the mean length of stay.

The secondary data base consists of census information taken from a demographic survey on May 18, 1977, from the five psychiatric wards at the Tampa VA Hospital. Because most of the patients were not discharged on the day of the survey, the length of stay statistics are incomplete or "censored" by the time of observation. This situation is analogous to Type II censoring in reliability studies. We observe a sample of n patients until r, a fixed number, are discharged. The random lengths of stay of the remaining n-r patients are recorded as the total time each has spent in the ward, up to the discharge of the rth patient.

2. General Problem

The standard hospital procedure for compiling census statistics is to have a medical records clerk, once a month, collate all the relevant data pertaining to patients admitted or discharged in the previous month. The mean length of stay is computed for the latter. Now some, indeed many, of the patients may have been discharged near the beginning of the previous month. Thus, there is at least a month's time lag involved before the results become known. Furthermore, it is not customary to compute the variance or study any of the statistical properties of the distribution of lengths of stay.

On the other hand, such information would be useful in the administration of a ward or hospital. A medical director may wish to schedule admissions or discharges with a view toward maintaining the census near some equilibrium. If it is known in advance that there is likely to be a temporary decline in bed occupancy, personnel may be assigned to other duties. Conversely, during a period of high occupancy, certain non-vital tasks, like routine maintenance, may be postponed, or non-emergency admissions may be deferred until the beds are available.

In general, the random variable represented by length of stay is influenced by the type of ward, admission diagnosis, and age of the patient. In a medical or surgical ward, for example, the mean and standard deviation are commonly on the order of a few days to a week. At a voluntary admission, private care psychiatric facility like St. Joseph's, however, the typical adult stays two to three weeks. An adolescent or child, however, may stay an average of one or more months, with a corresponding increase in the standard deviation. The extreme case occurs in public, custodial institutions like state mental hospitals, where patients may be warehoused essentially for an indefinite period.

In the next section, we fit an exponential probability density function to the St. Joseph's data, in order to determine a theoretical model for further study. This is followed by a discussion of appropriate estimators for a censored sample, along with an example taken from the VA Hospital data. This latter type of sample is suggested as a means of computing a more timely estimate of the mean length of stay.

3. Exponential Model

Previous authors have considered the problem of fitting a density function to observed hospital lengths of stay. Cooper and Cocoran (1974) used an exponential distribution, and DuFour (1974) used a lognormal. In general, lengths of stay tend to be unimodal with a strong rightward skew. The mode may appear at or near 1 day, giving the histogram a J-shaped appearance. Thus, either distribution would appear as likely candidates, as well as might various forms of the gamma or Weibull distributions.

Table 1 reproduces the length of stay frequencies, with the data grouped into 25 classes, for 1,310 patients at St. Joseph's Hospital. (Note that we are not considering patients who did not occupy a bed for at least 1 night). Because of the difference in distributions between adolescents and adults, and because we wished to generalize the model to the VA Hospital data, only patients aged 20 or more were considered. The resulting sample of patients had a mean length of stay of $\overline{x} = 17.044$ days, with a standard deviation of 22.241 days. Some age effect, however, still remains. Out of 21 patients with a recorded length of stay greater than 80 days, 14 were 20-24 years old. The distribution of ages of adult patients at St. Joseph's is more nearly uniform, with a range from 20 to 90 years.

A goodness-of-fit test for the exponential distribution

$$F(t) = 1 - \exp(-t/\beta) ,$$

where the mean β is estimated by $\bar{x}$, resulted in a value of $X^2 = 36.1$, $p = .042$ with 23 degrees of freedom. The "messiness" in the data mentioned above at least partially contributed to a too-heavy tail. Furthermore, the patients themselves have a broad heterogeneity of background and diagnosis in comparison to those at the VA Hospital. Thus, we are willing to accept this otherwise marginal value and assume that the data is exponential. Neither the lognormal, Weibull, or gamma distributions produced acceptable fits.

Table 1: Lengths of Stay for 1,310 Patients

Time in Days	Observed	Expected
< 3.5	244	243.19
3.5 - 6.5	181	172.18
6.5 - 9.5	163	144.39
9.5 - 12.5	119	121.09
12.5 - 15.5	108	101.54
15.5 - 18.5	82	85.155
18.5 - 21.5	82	71.411
21.5 - 24.5	76	59.886
24.5 - 27.5	35	50.221
27.5 - 30.5	42	42.115
30.5 - 33.5	33	35.318
33.5 - 36.5	16	29.618
36.5 - 39.5	15	24.838
39.5 - 42.5	16	20.829
42.5 - 45.5	17	17.467
45.5 - 48.5	12	14.648
48.5 - 51.5	6	12.284
51.5 - 54.5	8	10.301
54.5 - 57.5	8	8.6389
57.5 - 60.5	6	7.2446
60.5 - 63.5	3	6.0754
63.5 - 66.5	3	5.0948
66.5 - 69.5	4	4.2726
69.5 - 72.5	4	3.583
> 72.5	27	18.617

The probability density function of an exponential distribution is given by

$$f(t) = (1/\beta) \exp(-t/\beta) ,$$

with $E(t) = \beta$ and $Var(t) = \beta^2$. One interesting property for this distribution is that the hazard rate

$$h(t) = f(t)/(1 - F(t)) = 1/\beta$$

is a constant. The quantity $h(t)dt$ is the probability that a patient on the ward for t days will be discharged during the interval $(t, t+dt)$. For a long-term care, custodial institution it may be more realistic to fit a distribution that has a hazard rate which approaches 0 for large t. Either a lognormal or a Weibull density would fit this criterion.

4. Censored Sampling

For a complete sample, $\hat{\beta} = \bar{x}$ is an unbiased, maximum likelihood estimator for β. Let us assume a general case of censored sampling, where the sampling is progressive. We have a random sample of size n, where r patients are discharged at times $t_i (i = 1,\ldots,r)$, and n-r patients remain on ward at times t_{r_i} $(i = 1,\ldots,n-r)$. Then the likelihood of the sample is given by

$$L = \frac{n!}{(n-r)!} \prod_1^r (1/\beta) \exp(-t_i/\beta)$$

$$\times \prod_1^{n-r} (1 - F(t_{r_i})) .$$

Thus,

$$\ell n\ L = -r\ \ell n\ \beta - (1/\beta) \sum_1^r t_i$$

$$- (1/\beta) \sum_1^{n-r} t_{r_i} + \text{constant} .$$

and

$$\frac{\partial\ \ell n\ L}{\partial\ \beta} = -r/\beta + (1/\beta^2) \sum_1^r t_i + (1/\beta^2) \sum_1^{n-r} t_{r_i} .$$

The maximum likelihood estimator is given by

$$\hat{\beta} = (\sum_1^r t_i + \sum_1^{n-r} t_{r_i})/r .$$

Mann, Schafer, and Singpurwalla (1974) also derive a best invariant estimator,

$$\tilde{\beta} = r\hat{\beta} / (r+1) .$$

The following numbers are the lengths of stay for 27 patients from Ward 2 of the Tampa VA Hospital, on May 18, 1977. The first 4 patients (reading left to right) were discharged that day; the remaining 23 observations are censored.

2	19	1	5	60	18	25	47	88
87	78	76	38	63	57	57	53	51
45	34	29	28	28	22	11	15	12

Thus,

$$\hat{\beta} = 27/4 + 1022/4 = 262.25 ,$$

and

$$\tilde{\beta} = (4/5)\hat{\beta} = 209.8 .$$

In practice, if there is a high mean length of stay and low daily turnover (admissions and discharges), one may find that this procedure leads to an unsatisfactorily great amount of censoring. An alternative scheme would be to follow the selected group of patients over a specific a amount of time (say, one or two weeks). The algebraic results would be identical to those shown above. This procedure will have the advantage of reducing the number of observations which are censored, though at the cost of some delay in obtaining the desired results. The study period might be varied relative to the type of ward and what is known a priori about the mean length of stay.

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6. References

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